Instructions: Complete each of the following exercises for practice.

1. Compute the maximum rate of change of f at point P, and the direction in which it occurs.
   (a) $f(x,y) = 4y\sqrt{x}$; $P = (4,1)$
   (b) $f(x,y,z) = x\ln(yz)$; $P = (1,2,\frac{1}{2})$
   (c) $f(x,y,z) = \arctan(xyz)$; $P = (1,2,1)$

2. Compute the local maxima, local minima, and saddle points of the function.
   (a) $f(x,y) = x^2 + xy + y^2 + y$
   $f(x,y) = y(e^x - 1)$
   (b) $f(x,y) = x^2 + y^4 + 2xy$
   $f(x,y) = x^4 - 2x^2 + y^3 - 3y$
   (c) $f(x,y) = y\cos(x)$
   (d) $f(x,y) = xy + e^{-xy}$
   (e) $f(x,y) = xye^{-\frac{x^2+y^2}{2}}$
   (f) $f(x,y) = \sin(x)\sin(y)$

3. Find the absolute maximum and minimum values of the function f on the region R.
   (a) $f(x,y) = x^2 + y^2 - 2x$ on R the closed triangular region with vertices $(2,0)$, $(0,2)$, and $(0,-2)$
   (b) $f(x,y) = x + y - xy$ on R the closed triangular region with vertices $(0,0)$, $(0,2)$, and $(4,0)$
   (c) $f(x,y) = x^2 + 2y^2 - 2x - 4y + 1$ on $R = \{(x,y) : 0 \leq x \leq 2,\ 0 \leq y \leq 3\}$
   (d) $f(x,y) = xy^2$ on $R = \{(x,y) : 0 \leq x,\ 0 \leq y,\ x^2 + y^2 \leq 3\}$
   (e) $f(x,y) = 2x^3 + y^4$ on $R = \{(x,y) : x^2 + y^2 \leq 1\}$

4. Find the points on the cone $z^2 = x^2 + y^2$ closest to $(4,2,0)$.

5. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes (i.e. the planes $x = 0$, $y = 0$, and $z = 0$) and with one vertex in the plane $x + 2y + 3z = 6$.

6. Find the maximum volume of a rectangular box which can be inscribed in a sphere of radius r.

7. Find the dimensions of a lidless cardboard box with volume $32,000\ \text{cm}^3$ and minimizing the used cardboard.

8. What is the maximum volume of a rectangular box with diagonal length d?